Ör!

$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ matrisinin köşegenleştirilebilir olduğunu gösteriniz ve $P^{-1}AP = D$ köşegen matris olacak şekilde bir P regüler matrisi bulunuz.

Önce karakteristik değerlerini bulalım.

matrisin karakteristik polinomu $P_\lambda(A) = |\lambda I - A| = 0$

$$|\lambda I - A| = \begin{vmatrix} \lambda & -1 & -1 \\ -1 & \lambda & -1 \\ -1 & -1 & \lambda \end{vmatrix} = (\lambda-2)(\lambda+1)^2$$

$$\lambda_{1,2} = -1 \quad \lambda = 2$$

$\lambda = -1$ için karakteristik vektör bulalım.

$A(\alpha) = \lambda\alpha \Rightarrow A(\alpha) = -\alpha \qquad \alpha = (\alpha_1, \alpha_2, \alpha_3)$

karakteristik vektörleri

$\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ dir. $\mathbb{R}^3$ ün bazı olduğundan A köşegen.

P regüler matrisi bu karakteristik vektörü sütun olarak yazdığımız matris old. dan

$$P = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$ dir. Gerçekten

$$P^{-1}AP = D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$ dir.

Sorular

1) $x+2y-z=-4$
$x+y+z=3$
$2x-2y+z=7$

Lineer denklem sistemini katsayılar matrisinin tersini bularak çözünüz.

$Ax=B \Rightarrow X=A^{-1}B$ A nın tersini bulalım.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 2 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \approx \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 1 & 0 \\ 0 & -6 & 3 & -2 & 0 & 1 \end{array}\right]$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & -6 & 3 & -2 & 0 & 1 \end{array}\right] \approx \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & -1 & 2 & 0 \\ 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & -9 & 4 & -6 & 1 \end{array}\right]$$

$$\approx \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & -1 & 2 & 0 \\ 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{4}{9} & \frac{2}{3} & -\frac{1}{9} \end{array}\right] \approx \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & 0 & 1/3 \\ 0 & 1 & 0 & 1/9 & 1/3 & -2/9 \\ 0 & 0 & 1 & -4/9 & 2/3 & -1/9 \end{array}\right]$$

$$X=A^{-1}B=\begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{3} & -\frac{2}{9} \\ -\frac{4}{9} & \frac{2}{3} & -\frac{1}{9} \end{bmatrix}\begin{bmatrix} -4 \\ 3 \\ 7 \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

Ör: $x-y+4z=-4$
$8x-3y-z=8$
$2x-y+z=0$

Cramer metodu ile çöz.

$$\begin{vmatrix} 1 & -1 & 4 \\ 8 & -3 & -1 \\ 2 & -1 & 1 \end{vmatrix}=\begin{vmatrix} 1 & -1 & 4 \\ 0 & 5 & -33 \\ 0 & 1 & -7 \end{vmatrix}=-2\neq 0$$

$$x=\frac{-8}{-2}=4 \quad y=\frac{-16}{-2}=8 \quad z=\frac{0}{-2}=0$$

Örnek: $2x_1 + x_2 - 3x_3 = 4$
$-x_1 + 2x_2 + 2x_3 = 6$
$3x_1 + x_2 - 3x_3 = 6$

LU ayrışımı ile çöz.
veya A matrisini L ve U ayrış. bul.

$$A = \begin{array}{ccc} \beta_1 & \beta_2 & \beta_3 \end{array}$$
$$A=\begin{bmatrix} 2 & 1 & -3 \\ -1 & 2 & 2 \\ 3 & 1 & -3 \end{bmatrix}$$

$E_1: \alpha_2 \to \alpha_2 + \frac{1}{2}\alpha_1$
$\alpha_3 \to \alpha_3 - \frac{3}{2}\alpha_1$
$E_2: \alpha_3 \to \alpha_3 + \frac{1}{5}\alpha_2$

Önce U'yu bulalım.

$$U=\begin{bmatrix} 2 & 1 & -3 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{3}{2} \end{bmatrix} \overset{E_2}{\approx} \begin{bmatrix} 2 & 1 & -3 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & \frac{8}{5} \end{bmatrix}$$

$E_1: \beta_1 \to \frac{1}{2}\beta_1$

$$L=\begin{bmatrix} 1 & 1 & -3 \\ -\frac{1}{2} & 2 & 2 \\ \frac{3}{2} & 1 & -3 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & \frac{5}{2} & \frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & \frac{1}{2} \\ \frac{3}{2} & -\frac{1}{5} & \frac{3}{2} \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{3}{2} & -\frac{1}{5} & \frac{8}{5} \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{3}{2} & -\frac{1}{5} & 1 \end{bmatrix}$$

$Ax = B \qquad A = LU$
$L(UX) = B \qquad \underline{UX = Z}$ olsun.

$$LZ = B \Rightarrow \underset{3\times3}{\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{3}{2} & -\frac{1}{5} & 1 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}} = \begin{bmatrix} 4 \\ 6 \\ 6 \end{bmatrix}$$

$z_1 = 4$
$-\frac{z_1}{2} + z_2 = 6$
$z_2 = 8$
$z_3 = \frac{8}{5}$

$$UX = Z \Rightarrow \begin{bmatrix} 2 & 1 & -3 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & \frac{8}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ \frac{8}{5} \end{bmatrix}$$

$\boxed{x_3 = 1}$

$\frac{5}{2}x_2 + \frac{x_3}{2} = 8 \quad \boxed{x_2 = 3}$

$8 - \frac{1}{2} \quad \frac{15}{2} = \frac{5}{2}x_2$

$2x_1 + x_2 - 3x_3 = 4 \quad x_1 = 2$

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Örnek: $x_1+2x_3-x_5=3$ lineer denk sist. çözünüz.

$x_2-x_3-x_4+x_5=1$

$x_2+2x_3+x_4-2x_5=2$

$$[A:B]=\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & -1 & 3 \\ 0 & 1 & -1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 1 & -2 & 2 \end{array}\right]$$

$$\approx \left[\begin{array}{ccccc|c} 1 & 0 & 0 & -4/3 & 1 & 7/3 \\ 0 & 1 & 0 & -1/3 & 0 & 4/3 \\ 0 & 0 & 1 & 2/3 & -1 & 1/3 \end{array}\right]$$

rank $[A:B]=$ rank $A=3$ ve $5-3=2$ parametreye bağlı sonsuz çözüm bulunur.

$x_4=s \quad x_5=t$

$x_1=\frac{4}{3}s-t+\frac{7}{3} \quad x_2=\frac{1}{3}s+\frac{4}{3}, \quad x_3=-\frac{2}{3}s+t+\frac{1}{3}$

Örnek: $A_1=\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, \quad A_2=\begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}, \quad A_3=\begin{bmatrix} 1 & -5 \\ -4 & 0 \end{bmatrix}$

matrislerinin lineer bağımlılıklarını araştırınız.

$x_1A_1+x_2A_2+x_3A_3=0 \quad \Rightarrow ? \; x_i=0 \;\; \forall i.$

$x_1+3x_2+x_3=0$

$2x_1-x_2-5x_3=0$

$3x_1+2x_2-4x_3=0$

$x_1+2x_2=0$

homojen denk. sist. elementer işlemler uygularsa

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & -7 & -7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

olur. Rank $A=2$ aşikar olmayan çözümleri var. $3-2=1$ parametreye bağlı.

$\exists x_i \neq 0$ olup $\{A_1, A_2, A_3\}$ lineer bağımlıdır.